VI *Curvarum Hyperbolicarum, æquationibus trium nominum utcunque definitarum, Quadratura generalis duplici Theoremate exhibita à Dº.* Samuele Klingenſtierna, *Profeſſ. Digniſſ. Math. in Acad.* Upſal, *& R. S. S. Communicante Dº.* Jacobo Stirling, *ejuſdem etiam Soc. Doctiſſ. S.*

N. B. *CURVÆ Hyperbolicæ, de quarum quadraturâ hic agitur ab Erud. Auctore, ad unum quaſi genus reducuntur, ex communi quâ gaudent proprietate, quantumvis obſcura ſit nec ſatis per ſe determinata. Ad hoc enim genus refertur, omnis curva, cujus ordinata datum efficit rectangulum cum recta, quæ ex tribus partibus neceſſario diverſis & ordine genitis conſtituitur. Diverſæ partes eſſe intelliguntur, quæ ex diverſis abſciſſæ poteſtatibus quomodocunque oriuntur; Ordine autem genitæ ſunt, ſi modo ab ima ad ſummam poteſtatem æquis gradibus aſcendant.*

Species igitur determinantur ac definiuntur ex gradibus Poteſtatum determinatis & definitis.

Primas & ſimpliciſſimas hujus generis (ad quas etiam ceteræ omnes ultimo reducuntur) Neutonus *ipſe primus ex datis Circuli & Hyperbolæ areis dimenſus eſt.*

Coteſius *deinde plures eſſe hujus generis Species, etiam in infinitum (ſecundum ordinem determinatum) progredientes detexit, quæ ad eandem quadra-*

turæ formam ac priores iſtæ & ſimpliciores reduci poſſint; idque fecit ope Theorematis cujuſdam novi de Inventione radicum æquationum binomialium, ex determinata quadam diviſione circumferentiæ Circuli in partes æquales; cujus Theorematis mentio facta eſt in Erudito ſuo Opere de Harmonia menſurarum.

Iiſdem veſtigiis inſiſtendo D. Moivræus *Theorema Coteſianum ulterius promovit ad inventionem radicum æquationum Trinomialium, idque adhibendo arcum circuli determinatæ magnitudinis vice circumferentiæ totius. Quo invento omnes hujus generis Species inter ſe commenſurabiles eſſe ſecundum rationem quadraturæ ſuæ ſtatim perſpexit, Methodumque tradidit in exquiſitis ſuis ſcriptis* Miſcellaneis *nuper editis, quâ perveniatur ad quadraturam unius cujus libet formæ ex datis Circuli & Hyperbolæ quadraturis.*

Ds. Kl. *in Propoſitione ſua, quæ ſequitur, in unum collegit quicquid de quadraturis curvarum hujus generis antehac a prioribus inventum fuit. Verum tamen ita collegit non quaſi ſint variæ formæ ſub uno genere, ſed quaſi una ſit eademque forma generis ipſius. Theorema duplex eſt, quatenus quadratura referat ad aream vel citra, vel ultra ordinatam. Exhibetur in ipſis æquationis terminis ſine reductione aut reſtrictione. Inſtituitur ſecundum* Coteſii *doctrinam, uſurpando menſuras Angulorum & Rationum pro areis Circuli & Hyperbolæ. Traditur ſine demonſtratione, utpote cujus veritas facile innoteſcat ex Propoſitionibus* Moivræanis.

Hæc de hujus doctrinæ fontibus indigitaſſe, non abs re fore judicatum eſt, ne, lectoribus inexercitatis, auctoris nimia brevitas impedimento eſſet.

PROPOSITIO.

Quadrare curvam, cujus abſciſſa eſt z & ordinata $\frac{cz^{\eta \pm \frac{r}{n}\eta - 1}}{a^{2n} \pm a^{n-1}bz^{n} + z^{2n}}$, ubi η deſignat numerum quemlibet, r & n numeros quoſlibet integros & primos inter ſe, & denominator $a^{2n} \pm a^{n-1}bz^{n} + z^{2n}$ non poteſt reſolvi in duos factores binomios.

In circumferentia circuli (*Tab.* 2. *Fig.* 1.) centro quovis O intervallo OR $= a$ deſcripta applicetur chorda RT $= b$, cui parallelus ducatur radius OP, ita quidem ut arcus PR ſit quadrante major ſi habeatur $+ b$, minor vero ſi habeatur $- b$. Incipiendo in puncto R, ſumantur ordine tot arcus $R\overset{\mathrm{i}}{R}$, $\overset{\mathrm{i}}{R}\overset{\mathrm{ii}}{R}$, $\overset{\mathrm{ii}}{R}\overset{\mathrm{iii}}{R}$, $\overset{\mathrm{iii}}{R}\overset{\mathrm{iv}}{R}$, $\overset{\mathrm{iv}}{R}\overset{\mathrm{v}}{R}$, &c. arcui PR æquales, quot unitates continet fractio $\frac{r}{n}$, & a punctis $\overset{\mathrm{i}}{R}$, $\overset{\mathrm{ii}}{R}$, $\overset{\mathrm{iii}}{R}$, $\overset{\mathrm{iv}}{R}$, $\overset{\mathrm{v}}{R}$, &c. ducantur totidem rectæ $\overset{\mathrm{i}}{R}\overset{\mathrm{i}}{r}$, $\overset{\mathrm{ii}}{R}\overset{\mathrm{ii}}{r}$, $\overset{\mathrm{iii}}{R}\overset{\mathrm{iii}}{r}$, $\overset{\mathrm{iv}}{R}\overset{\mathrm{iv}}{r}$, $\overset{\mathrm{v}}{R}\overset{\mathrm{v}}{r}$, &c. radio OP parallelæ & rectæ OR occurrentes in punctis, $\overset{\mathrm{i}}{r}$, $\overset{\mathrm{ii}}{r}$, $\overset{\mathrm{iii}}{r}$, $\overset{\mathrm{iv}}{r}$, $\overset{\mathrm{v}}{r}$, &c. Deinde dividatur arcus PR in tot partes æquales quot ſunt unitates in numero n, quarum illa quæ puncto P ad-

adjacet ſit P A. Facto initio in puncto A dividatur integra circumferentia in tot partes æquales A B, B C, C D, D E, &c. quot ſunt unitates in n; ſumtaque in radio O P, producto ſi opus ultra P, abſciſſa O S $= a.\overline{\frac{z}{a}}\Big|^{\frac{n}{n}}$, jungantur S A, S B, S C, S D, S E, &c. ut & O A, O B, O C, O D, O E, &c. Denique ſumantur arcus P A a, P B b, P C c, P D d, P E e. &c. qui ſint ad arcus P A, P B, P C, P D, P E, &c. ut $n + r$ ad unitatem, & a punctis a, b, c, d, e, &c. ducantur tum rectæ $a\alpha$, $b\beta$, $c\gamma$, $d\delta$, $e\varepsilon$, &c. parallelæ radio O P & occurrentes rectæ O R in punctis α, β, γ, δ, ε, &c. tum etiam rectæ a 1, b 2, c 3, d 4, e 5, &c. prioribus normales, & rectæ Q O, quæ ad R O ducatur perpendicularis, occurrentes in punctis 1, 2, 3, 4, 5, &c.

His factis area curvæ cujus abſciſſa eſt z & ordinata $\dfrac{cz^{n+\frac{r}{n}n-1}}{a^{2n} \pm a^{n-1}bz^{n} + z^{2n}}$, erit $\dfrac{nc}{na^{n+1}}z^{\frac{r}{n}n}$ in

$$\frac{\overset{\text{i}}{\mathrm{R}}\,\overset{\text{i}}{r}}{r-n}\times\overline{\frac{a}{z}}\Big|^{n} - \frac{\overset{\text{ii}}{\mathrm{R}}\,\overset{\text{ii}}{r}}{r-2n}\times\overline{\frac{a}{z}}\Big|^{2n} + \frac{\overset{\text{iii}}{\mathrm{R}}\,\overset{\text{iii}}{r}}{r-3n}\times\overline{\frac{a}{z}}\Big|^{3n}$$

$$- \frac{\overset{\text{iv}}{\mathrm{R}}\,\overset{\text{iv}}{r}}{r-4n}\times\overline{\frac{a}{z}}\Big|^{4n} - \frac{\overset{\text{v}}{\mathrm{R}}\,\overset{\text{v}}{r}}{r-5n}\times\overline{\frac{a}{z}}\Big|^{5n}, \text{ \&c.}$$

$$+\frac{c}{n}a^{\frac{r}{n}n-n-1} \text{ in} \left\{\begin{array}{l} -a\alpha\,(\mathrm{SA:AO}) - a1\,(+\mathrm{SAO}) \\ +b\beta\,(\mathrm{SB:BO}) + b2\,(+\mathrm{SBO}) \\ -c\gamma\,(\mathrm{SC:CO}) + c3\,(+\mathrm{SCO}) \\ +d\delta\,(\mathrm{SD:DO}) - d4\,(-\mathrm{SDO}) \\ +e\varepsilon\,(\mathrm{SE:EO}) + e5\,(-\mathrm{SEO}) \\ \qquad\quad \text{\&c.} \qquad\qquad\quad \text{\&c.} \end{array}\right\}$$

Et

Et area curvæ cujus abſciſſa eſt z & ordinata

$$\frac{c z^{n-\frac{r}{n}n-1}}{a^{2n} \pm a^{n-1} b z^{n} + z^{2n}}, \text{ erit } \frac{nc}{na^{n+1}} z^{-\frac{r}{n}n} \quad \text{in}$$

$$\frac{\overset{\mathrm{I}}{R}\,\overset{\mathrm{I}}{r}}{r-n} \times \left.\overline{\frac{z}{a}}\right|^{n} - \frac{\overset{\mathrm{II}}{R}\,\overset{\mathrm{II}}{r}}{r-2n} \times \left.\overline{\frac{z}{a}}\right|^{2n} + \frac{\overset{\mathrm{III}}{R}\,\overset{\mathrm{III}}{r}}{r-3n} \times \left.\overline{\frac{z}{a}}\right|^{3n}$$

$$- \frac{\overset{\mathrm{IV}}{R}\,\overset{\mathrm{IV}}{r}}{r-4n} \times \left.\overline{\frac{z}{a}}\right|^{4n} - \frac{\overset{\mathrm{V}}{R}\,\overset{\mathrm{V}}{r}}{r-5n} \times \left.\overline{\frac{z}{a}}\right|^{5n}, \text{ \&c.}$$

$$+ \frac{c}{n} a^{-\frac{r}{n}n-n-1} \text{ in } \left\{ \begin{array}{l} -a\alpha(SA:AO) - a1(+ASO) \\ +b\beta(SB:BO) + b2(+BSO) \\ -c\gamma(SC:CO) + c3(+CSO) \\ +d\delta(SD:DO) - d4(-DSO) \\ +e\varepsilon(SE:EO) + e5(-ESO) \\ \qquad\quad \text{\&c.} \qquad\qquad\quad \text{\&c.} \end{array} \right\}$$

Harum arearum prior adjacet abſciſſæ ad ordinatam terminatæ, poſterior vero abſciſſæ ultra ordinatam productæ. Signa autem quantitatum has expreſſiones ingredientium ita determinantur: 1. Rectæ $\overset{\mathrm{I}}{R}\,\overset{\mathrm{I}}{r}$, $\overset{\mathrm{II}}{R}\,\overset{\mathrm{II}}{r}$, $\overset{\mathrm{III}}{R}\,\overset{\mathrm{III}}{r}$, &c. afficiuntur ſignis affirmativis, ſi a punctis circumferentiæ $\overset{\mathrm{I}}{R}$, $\overset{\mathrm{II}}{R}$, $\overset{\mathrm{III}}{R}$, tendunt ſecundum directionem O P, negativis vero ſi ab iiſdem punctis ſecundum directionem contrariam P O procedunt. 2. Moduli rationum $a\alpha$, $b\beta$, $c\gamma$, &c. ſigna habent affirmativa, ſi a punctis a, b, c, &c. tendunt ſecundum directionem O P, negativa ſi ſecundum contrariam. 3. E centro circuli O cadat in chordam R T normalis O H. Et moduli angulorum *a* 1, *b* 2, *c* 3, *&c.* ſignis gaudebunt affirmativis ſi a punctis *a*, *b*, *c*, *&c.* tendunt ſecundum directionem H O, negativis ſi ſecundum contrariam. 4. Producatur radius P O donec circumferentiæ denuo occurrat in *p*, & anguli S A O, S B O, S C O, *&c.* ut & A S O, B S O, C S O, *&c.* ſumi debent affirmative ſi exiſtunt in ſemicirculo ſuperiore P R *p*, negative ſi in inferiore. Et ſecundum has regulas ſigna quantitatum quibus areæ exprimuntur noſtræ figuræ accommodavimus.

VII. *Caſus rariſſimus* Plicæ Polonicæ *enormis à D.* Abrahamo Vatero, *M. D. Prof. Anatom.* Wittemberg. *& R. S. S. per D.* Conradum Sprengell, *Equitem, M. D. R. S. S. & Coll. Med.* Lond. *Licent. communicatus.* Vid. Tab. II. Fig. 2.

FŒMINA ruſtica in Polonia, in terris Principis Radzivil, anno ætatis decimo quinto, viro nupta, incidit decimo octavo, in morbum Poloniæ Endemium, qui Plica Polonica a capillo inenodabili vocatur. Hanc Plicam per quinquaginta annos fœmina geſtavit, ac per totum fere illud tempus dolore arthritico et contracturis tandemque maraſmo univerſali corporis afflicta tecto affixa fuit, tandemque ſenio confecta anno ætatis ſeptuageſimo octavo diem ſuum obiit.

Philo: Trans: N.° 417.

TAB. II.

Fig. 2.

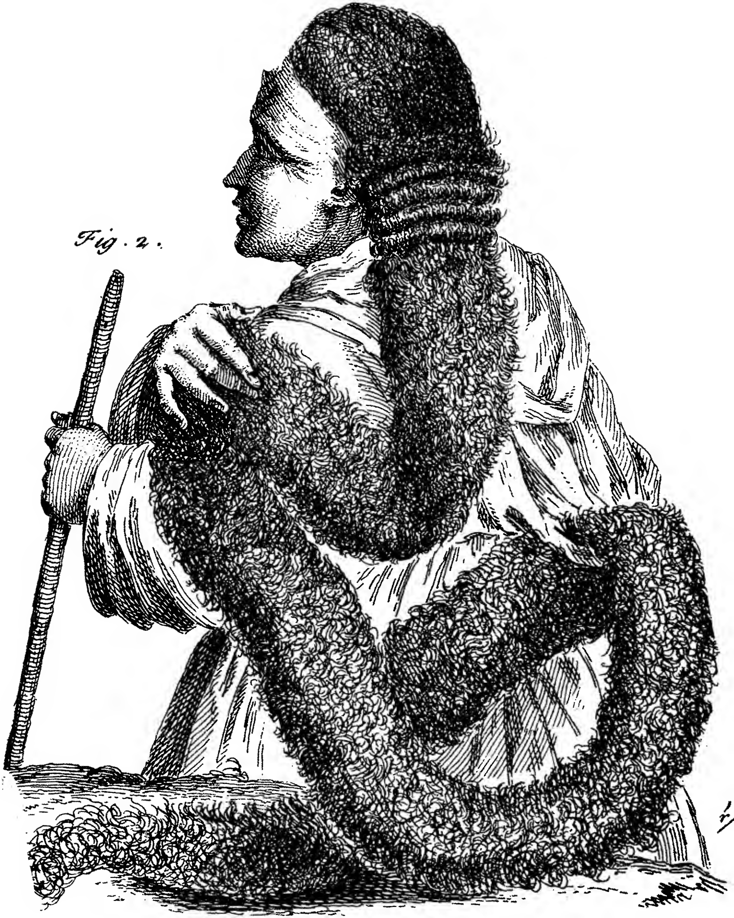

Fig. 1.